

Today: models

- More detail may mean less useful
- Dimension of a model
- Transition probabilities
- Jukes and Cantor (1969) model
- Long run behavior of a model
- Meet the other members of the family (F81, K80, F84, HKY, GTR)

A very *practical* MBTA subway map



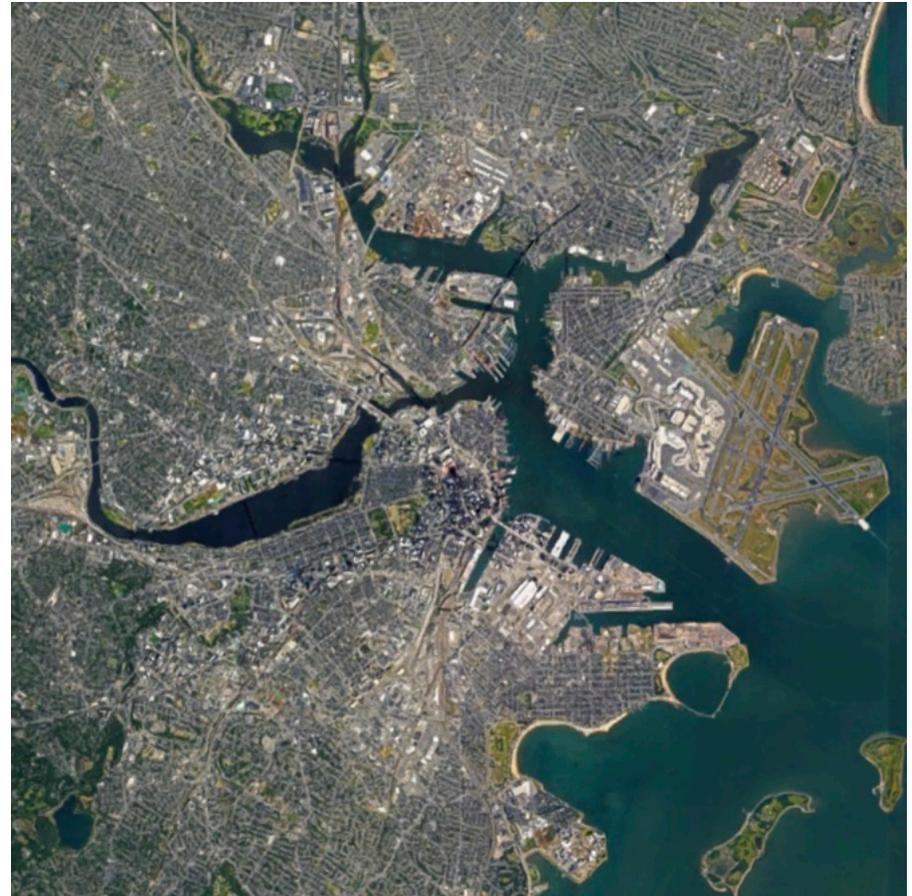
T...The Alternate Route.



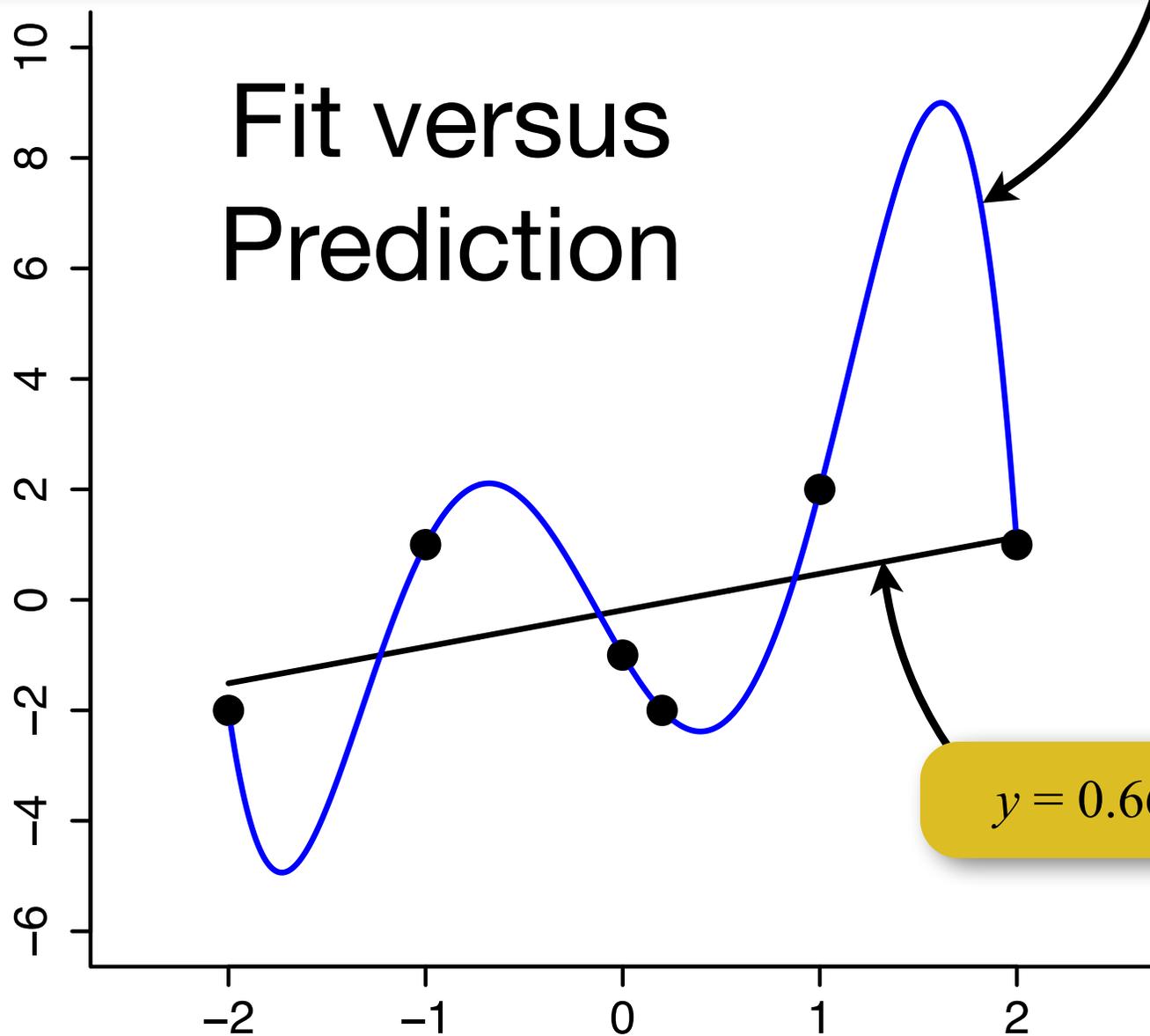
A very *realistic* MBTA subway map



Which is more useful?

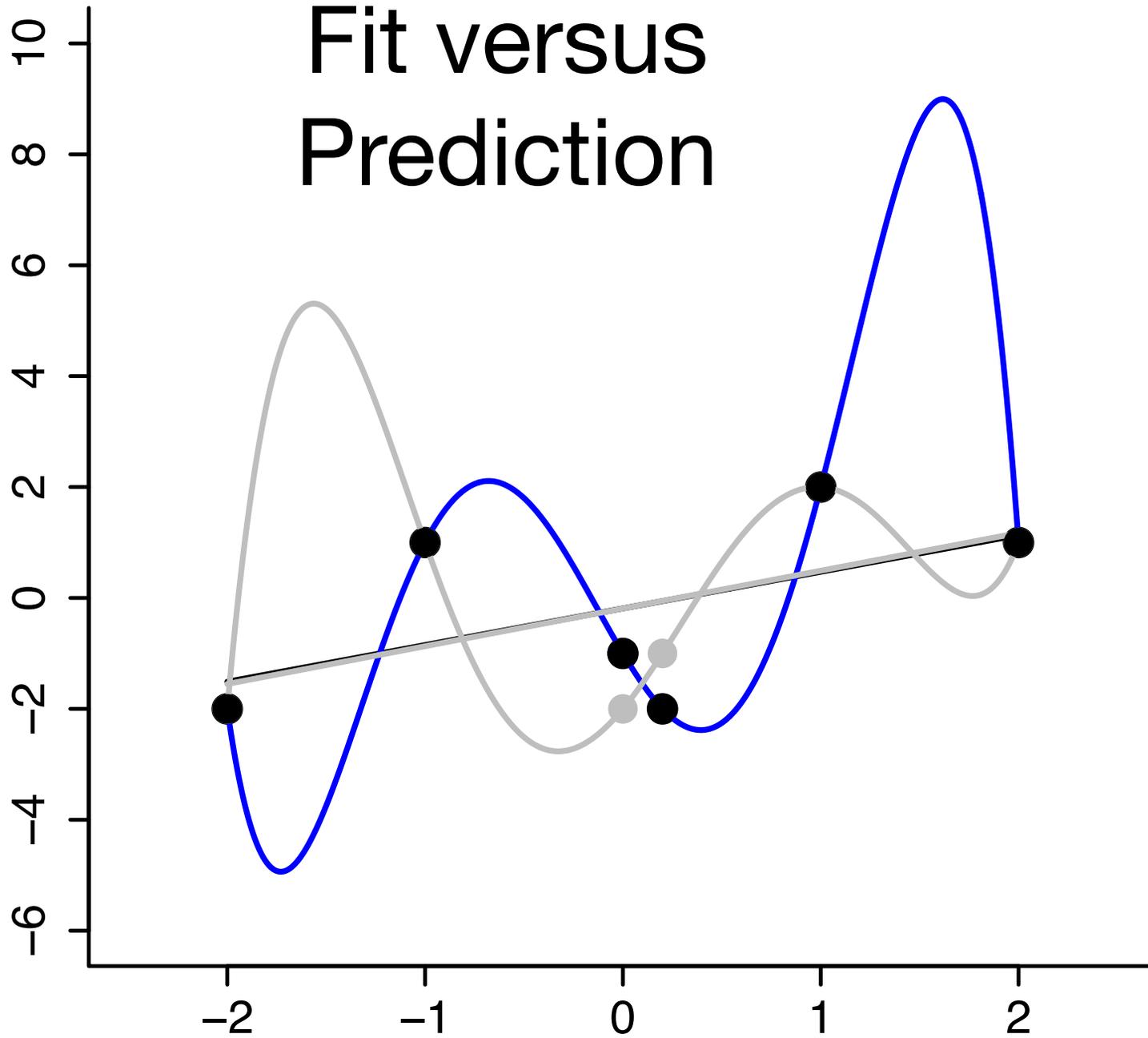


$$y = -1.5972 x^5 + -0.7917 x^4 + 8.0694 x^3 + 3.2917 x^2 + -5.9722 x + -1.0$$



$$y = 0.6611x + -0.1887$$

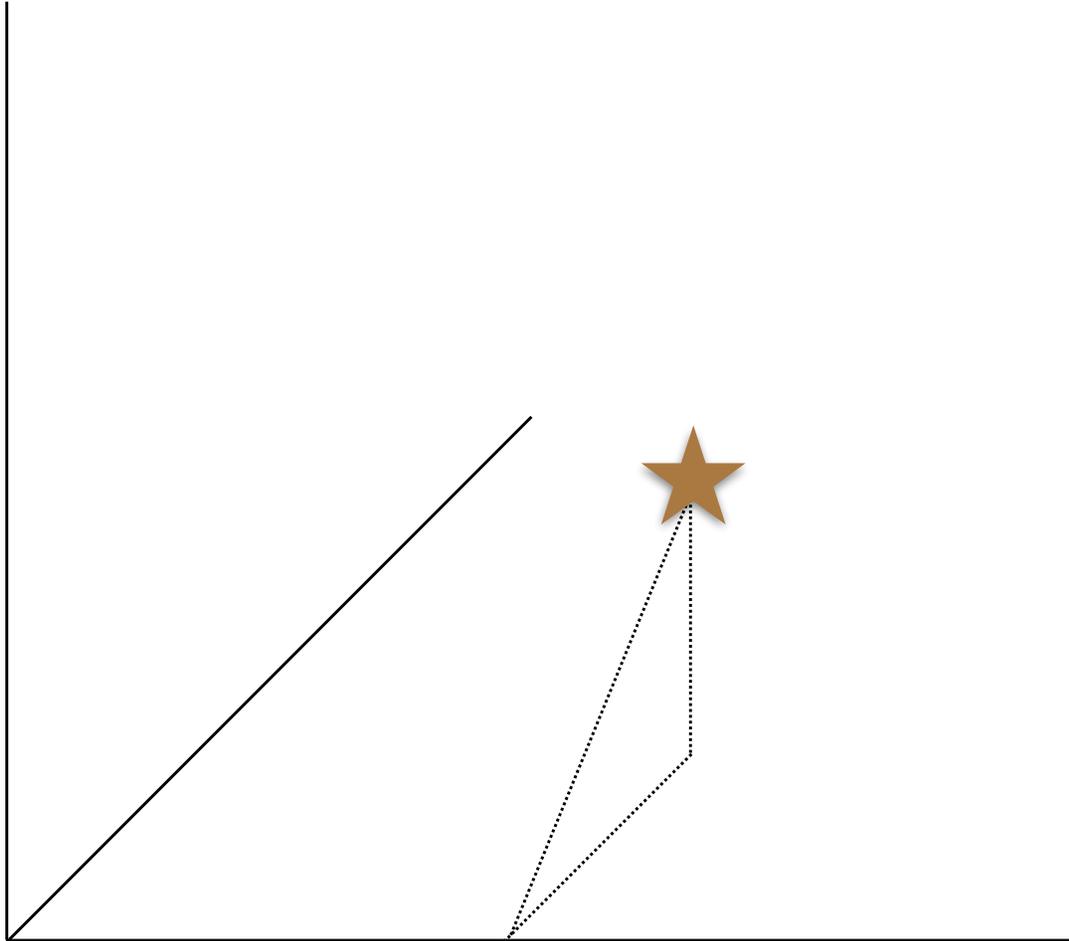
Fit versus Prediction



complex models

simple models

Model dimensions

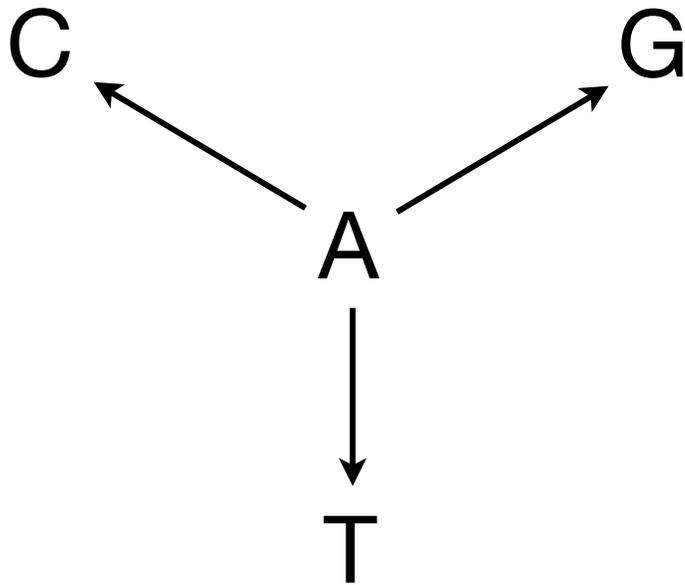


1-parameter model: 2.83

2-parameter model: 2.00

3-parameter model: 0.00

JC69 model



Jukes & Cantor (1969)

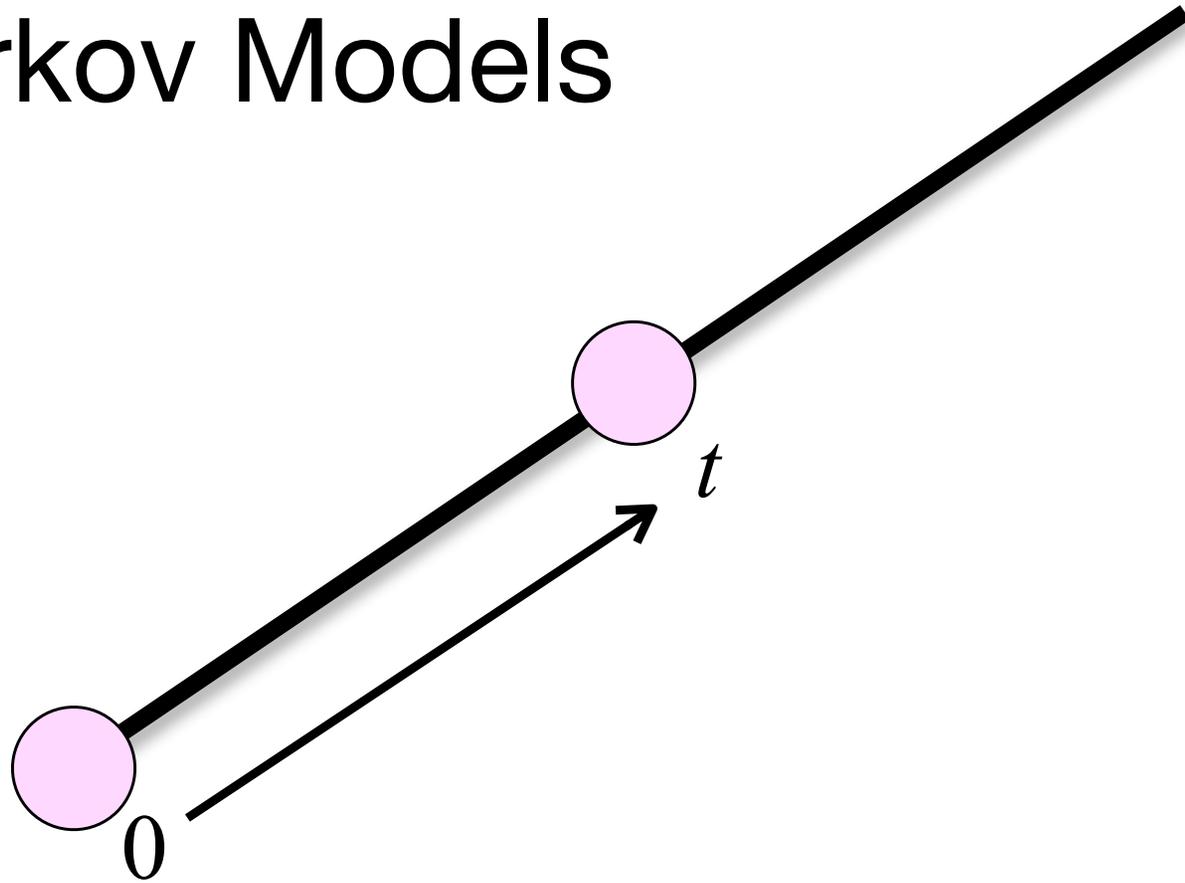
number of substitutions
substitution rate
total substitution rate

Edge lengths

edge length parameters

long edge lengths means...

Markov Models



Markov property
transition probability
conditional probability
multiple hits

Evolutionary Distance



JC69 Transition Probability

$$P_{TG}(t) = \frac{1}{4} (1 - e^{-4\beta t})$$

e = 2.718281828459045...

achnow rate vs. substitution rate

poisson distribution

JC69 Transition Probability

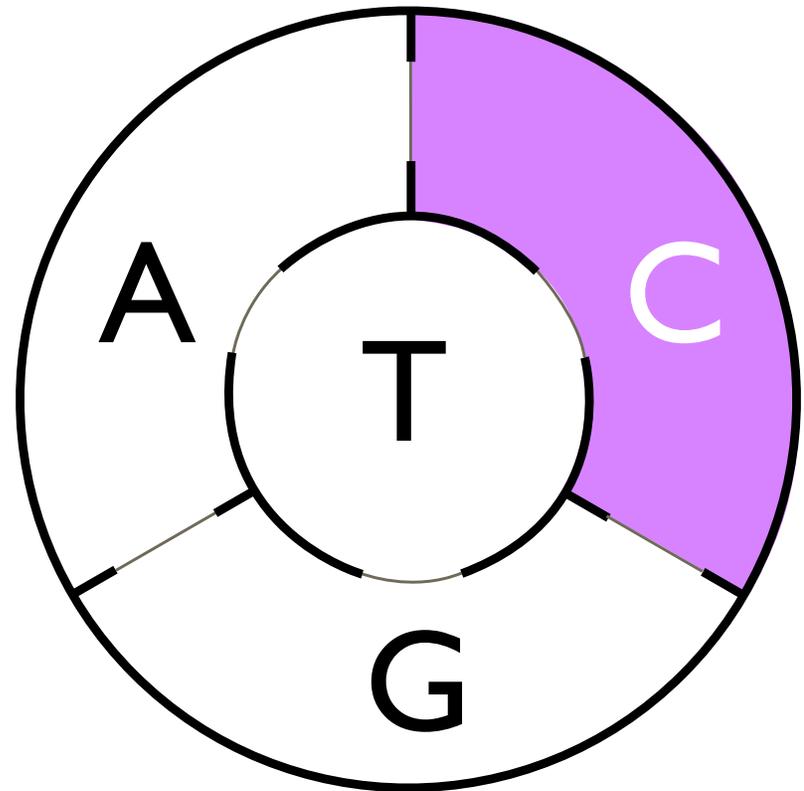
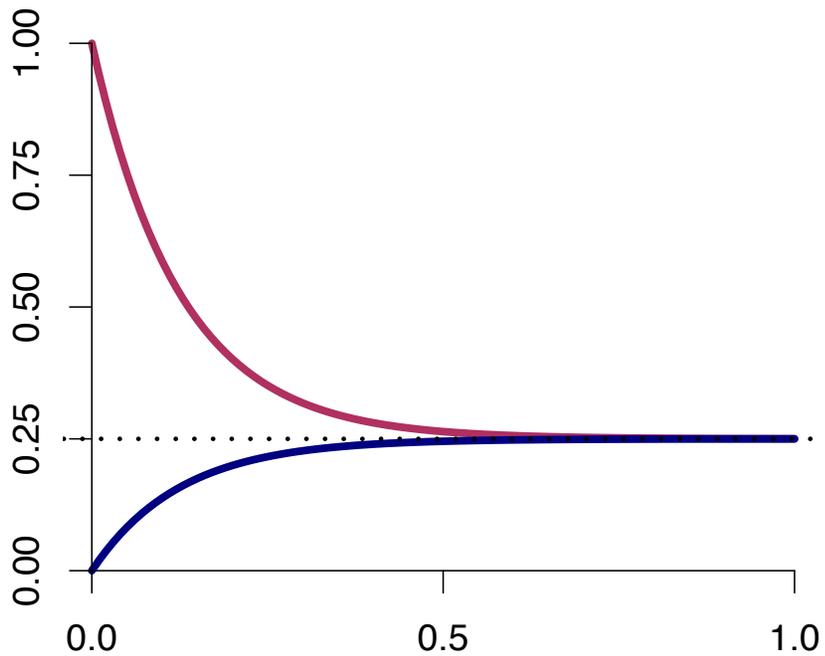
$$P_{TA}(t) = \frac{1}{4} - \frac{1}{4}e^{-4\beta t}$$

$$P_{TC}(t) = \frac{1}{4} - \frac{1}{4}e^{-4\beta t}$$

$$P_{TG}(t) = \frac{1}{4} - \frac{1}{4}e^{-4\beta t}$$

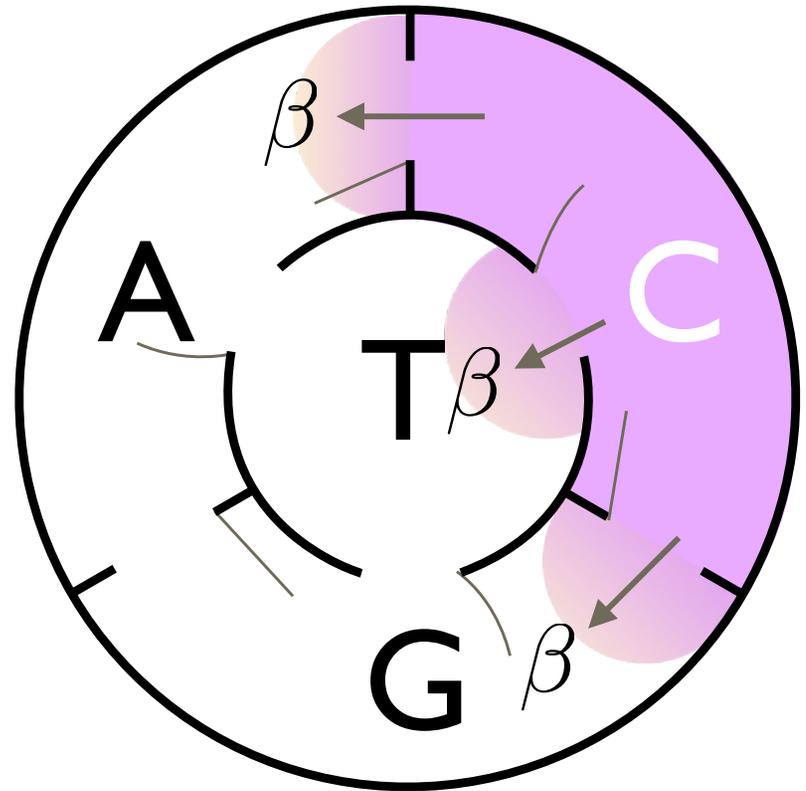
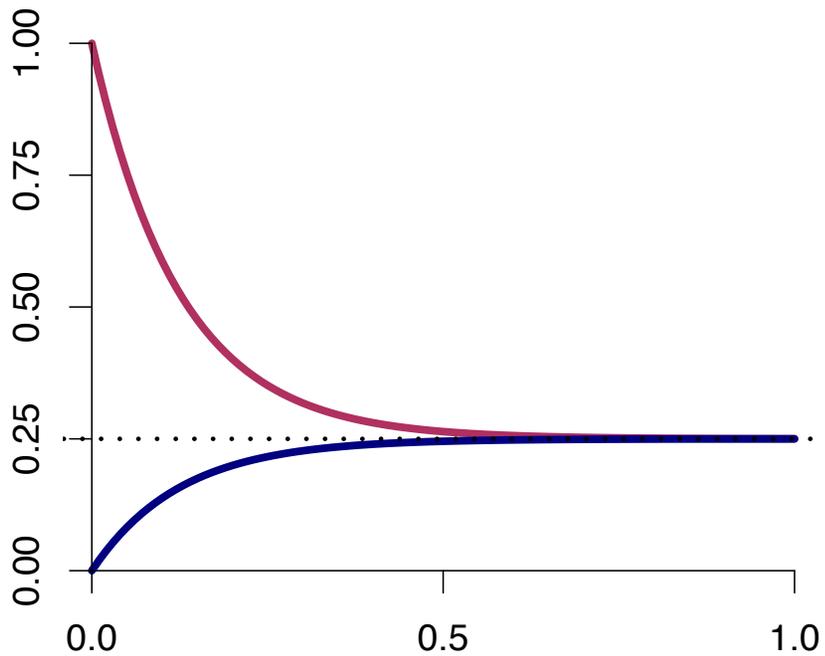
$$P_{TT}(t) = \frac{1}{4} - \frac{1}{4}e^{-4\beta t}$$

Equilibrium Frequencies

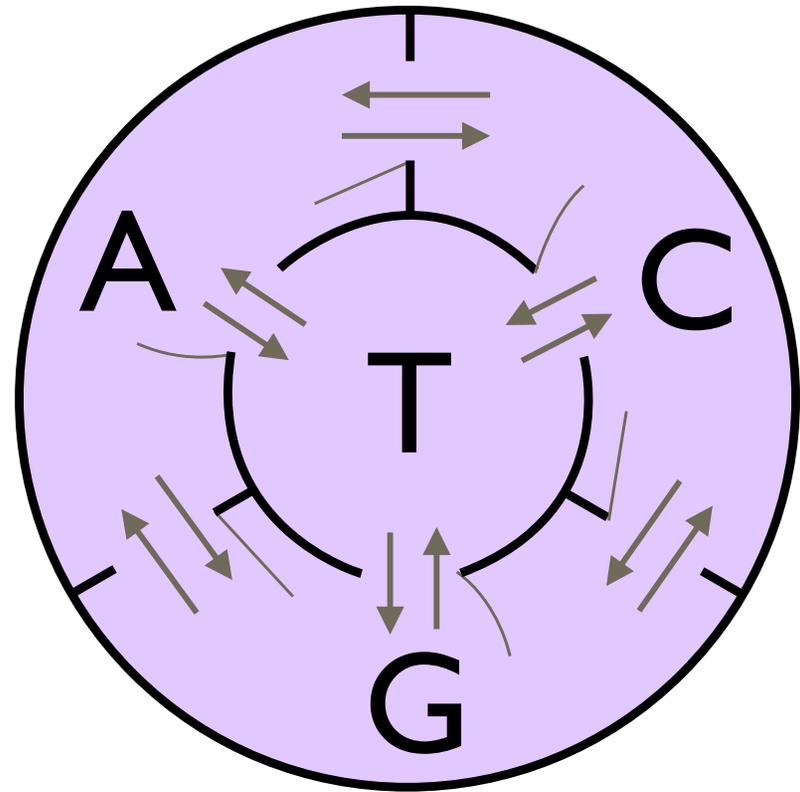
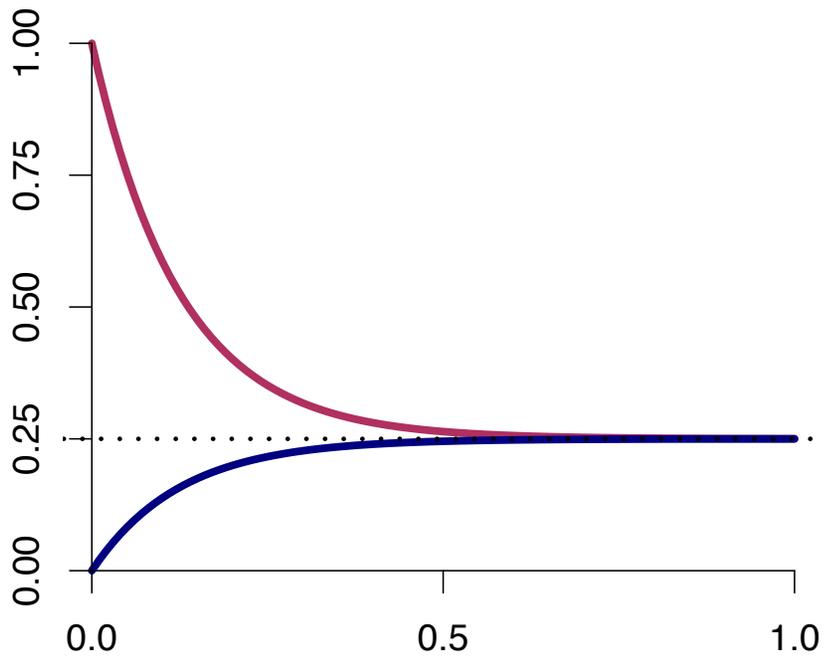


(architect: Joe Bielawski)

Equilibrium Frequencies



Equilibrium Frequencies



$$\pi_A = \pi_C = \pi_G = \pi_T = \frac{1}{4}$$

JC69 instantaneous rate matrix

		“To” state			
		A	C	G	T
“From” state	A				
	C				
	G				
	T				

K80 (K2P) rate matrix

		"To" state			
		A	C	G	T
"From" state	A				
	C				
	G				
	T				

rate of transversions
transition/transversion rate ratio
parameters?
equivalence to JC69