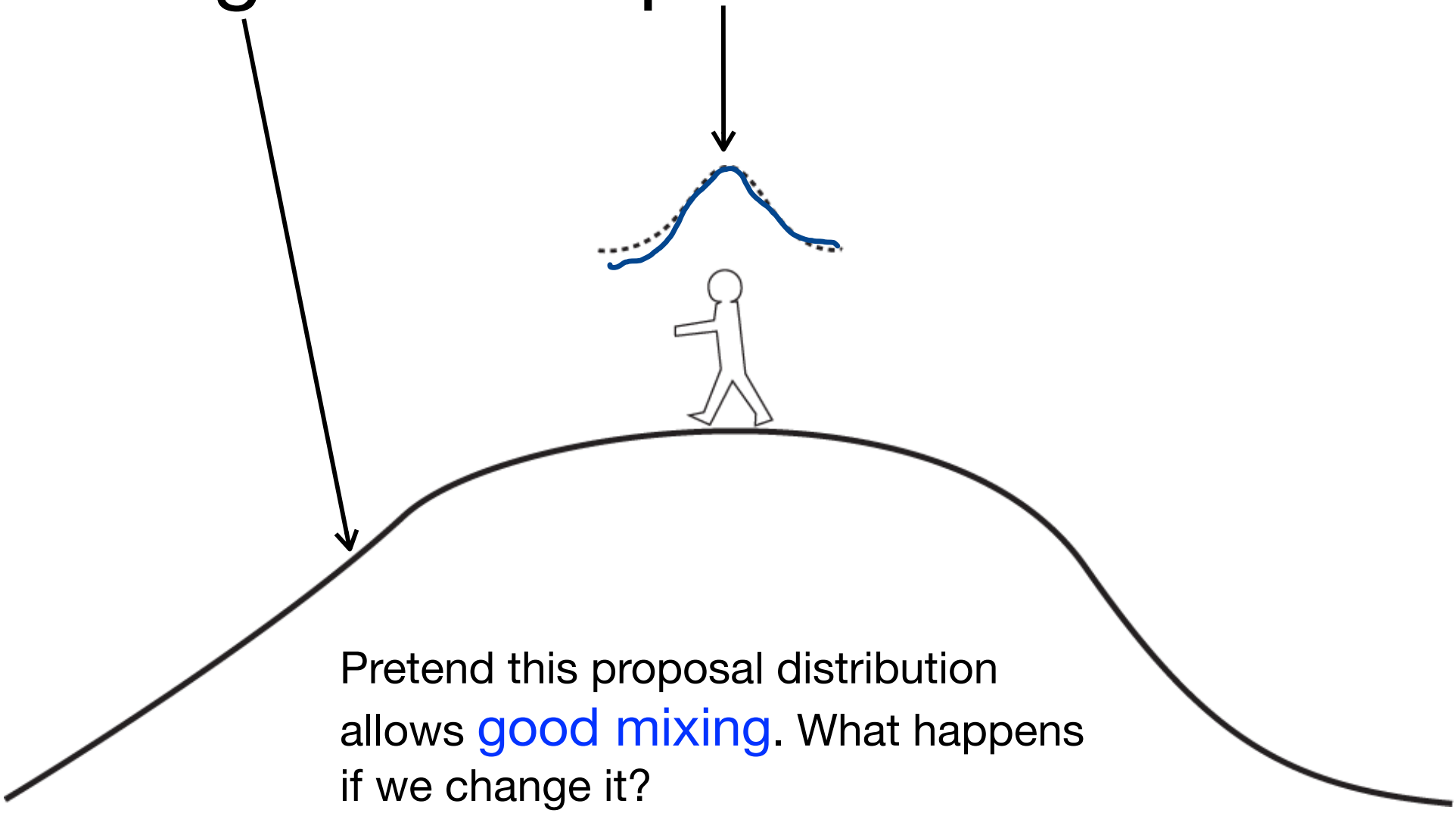
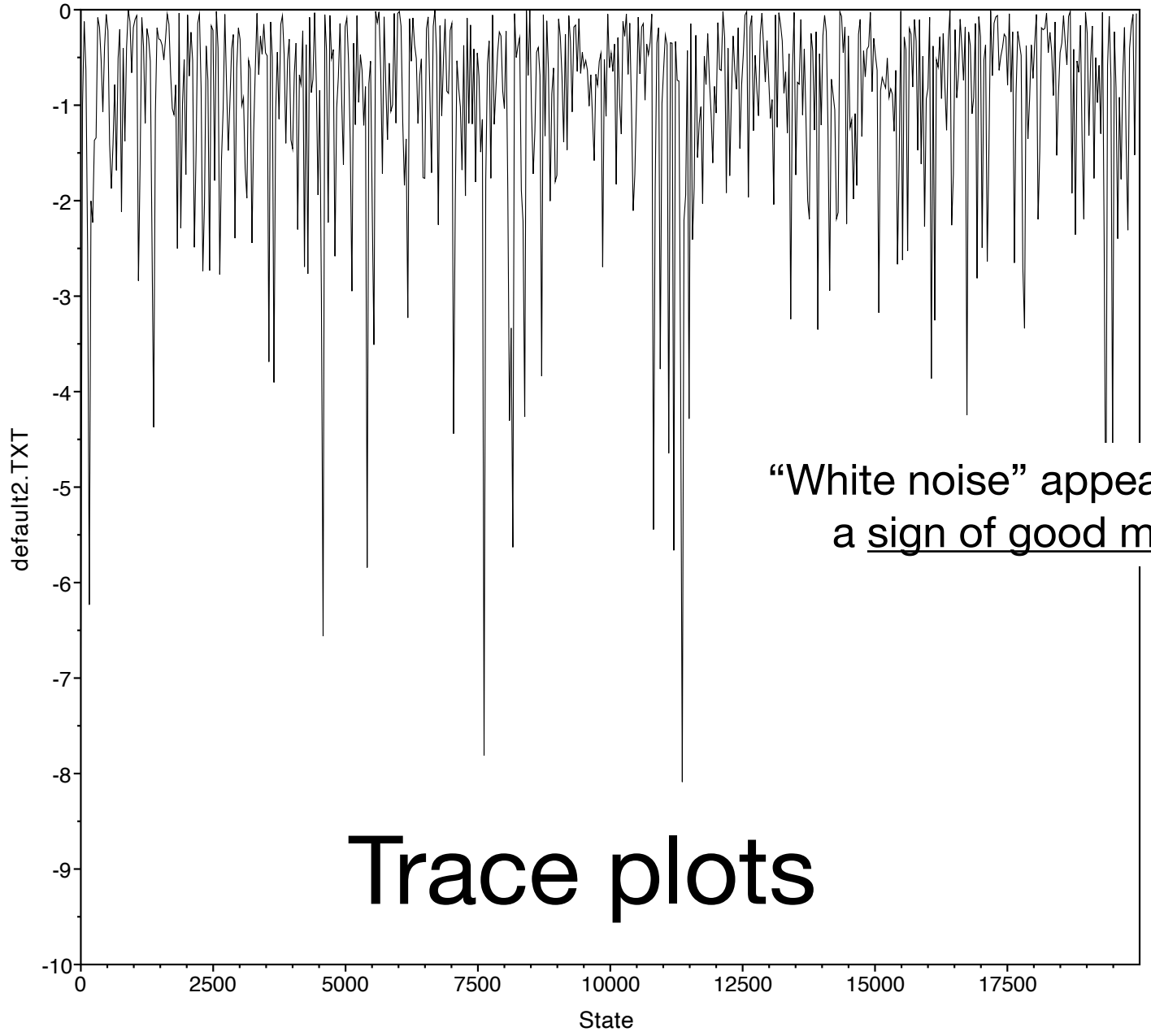
Target vs. Proposal Distributions



Pretend this proposal distribution allows good mixing. What happens if we change it?



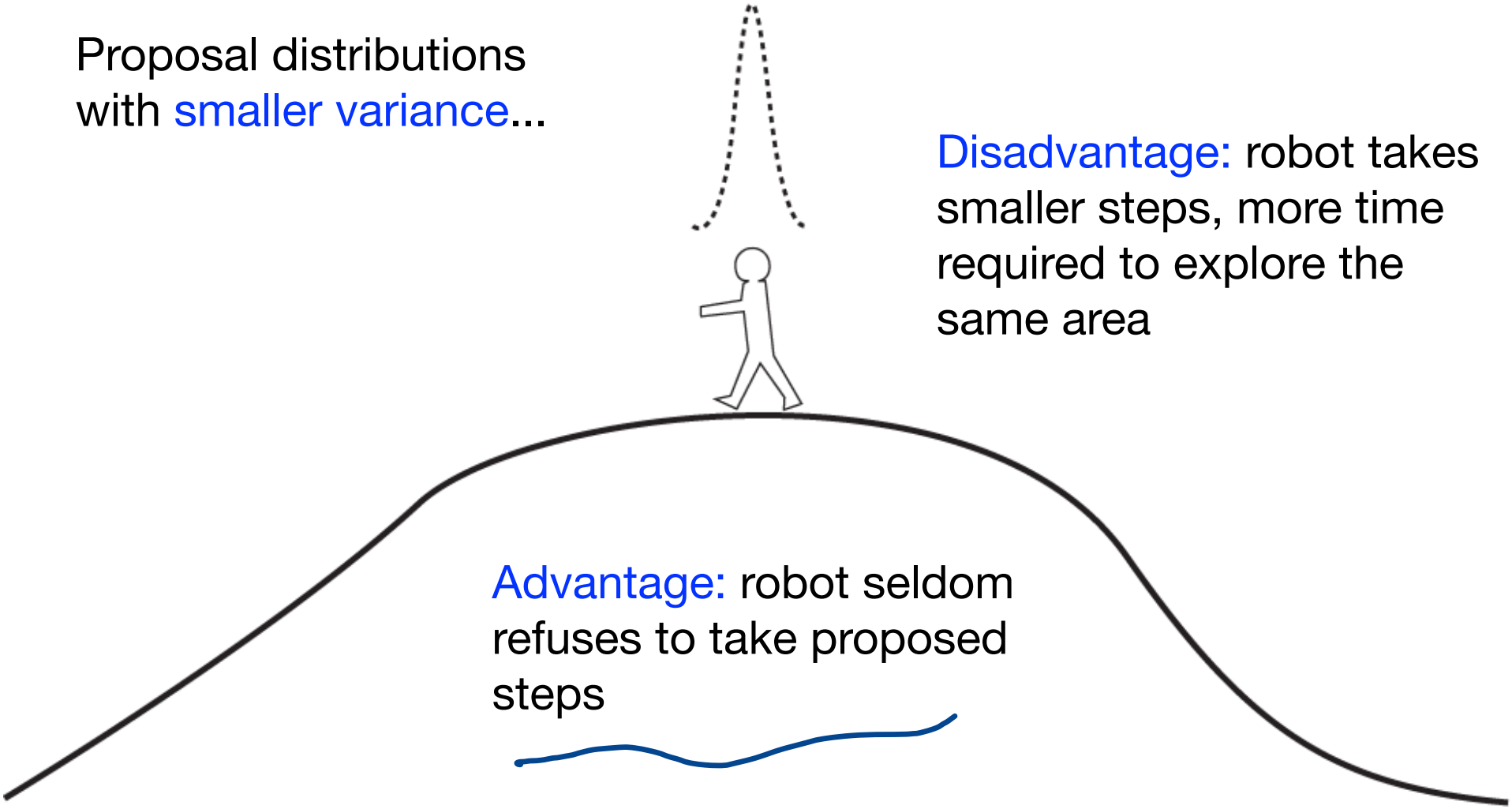
Trace plots

"White noise" appearance is a sign of good mixing

I used the program Tracer to create this plot:
http://tree.bio.ed.ac.uk/software/tracer/

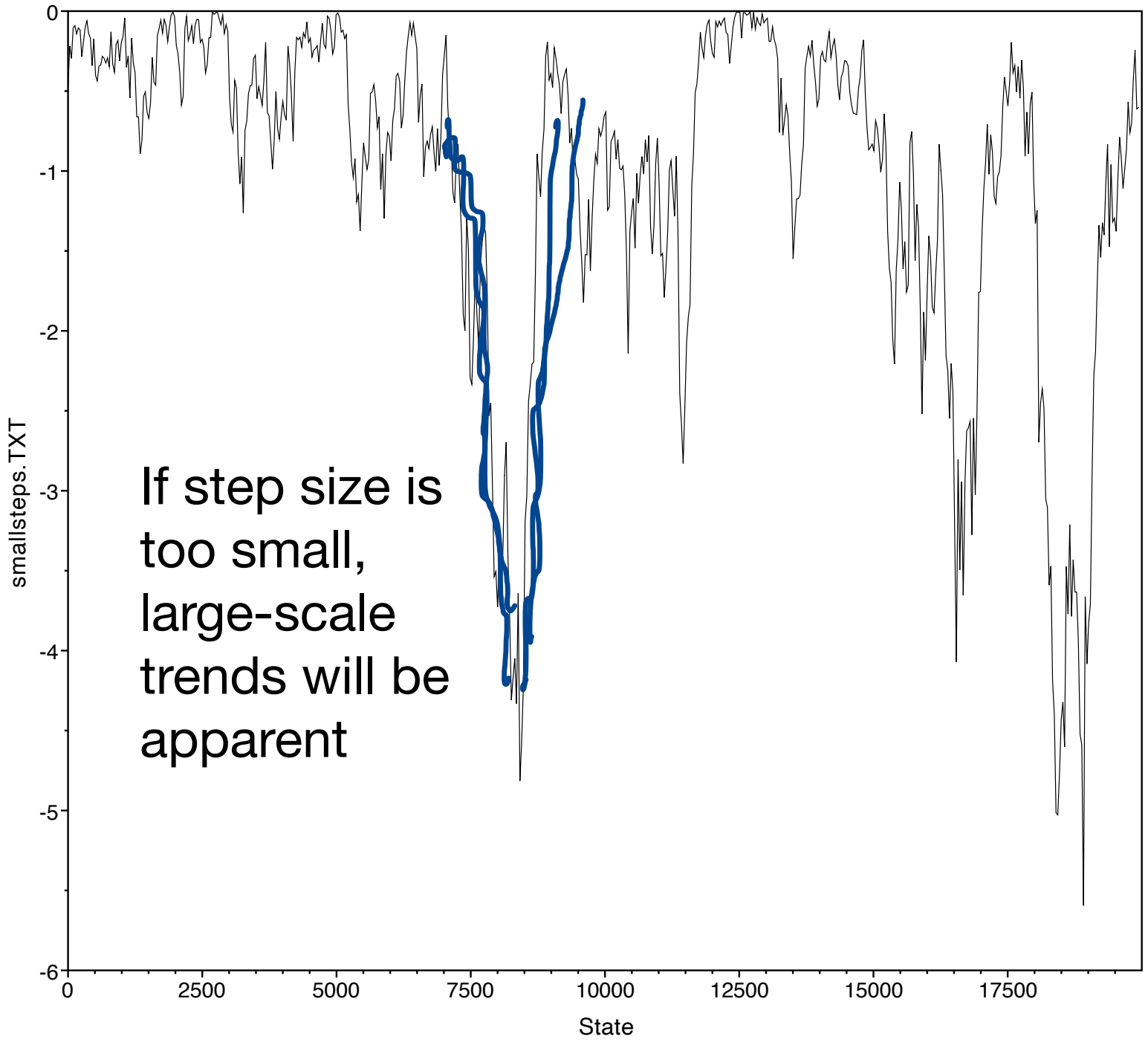
Target vs. Proposal Distributions

Proposal distributions with smaller variance...



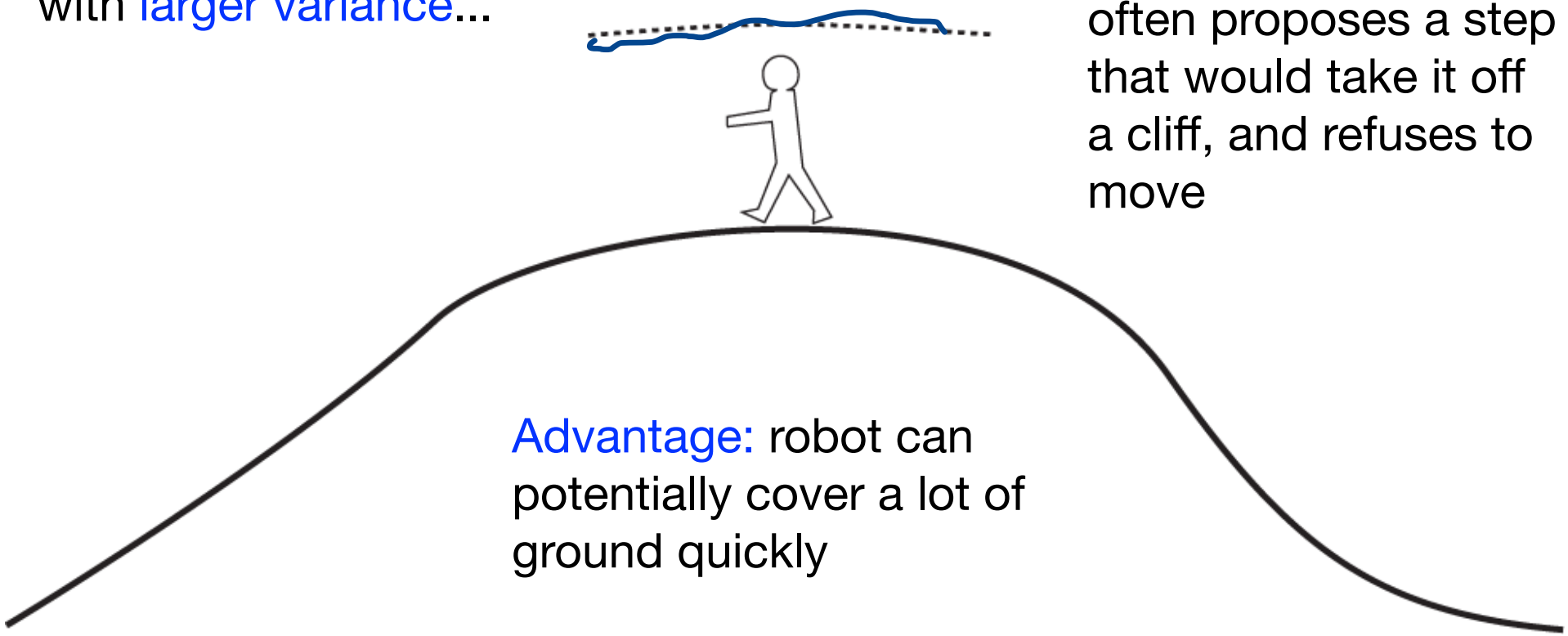
Disadvantage: robot takes smaller steps, more time required to explore the same area

Advantage: robot seldom refuses to take proposed steps


0
-1
-2
-3
-4
-5
-6
smallsteps.TXT
0
2500
5000
7500
10000
12500
15000
17500
State
If step size is too small, large-scale trends will be apparent

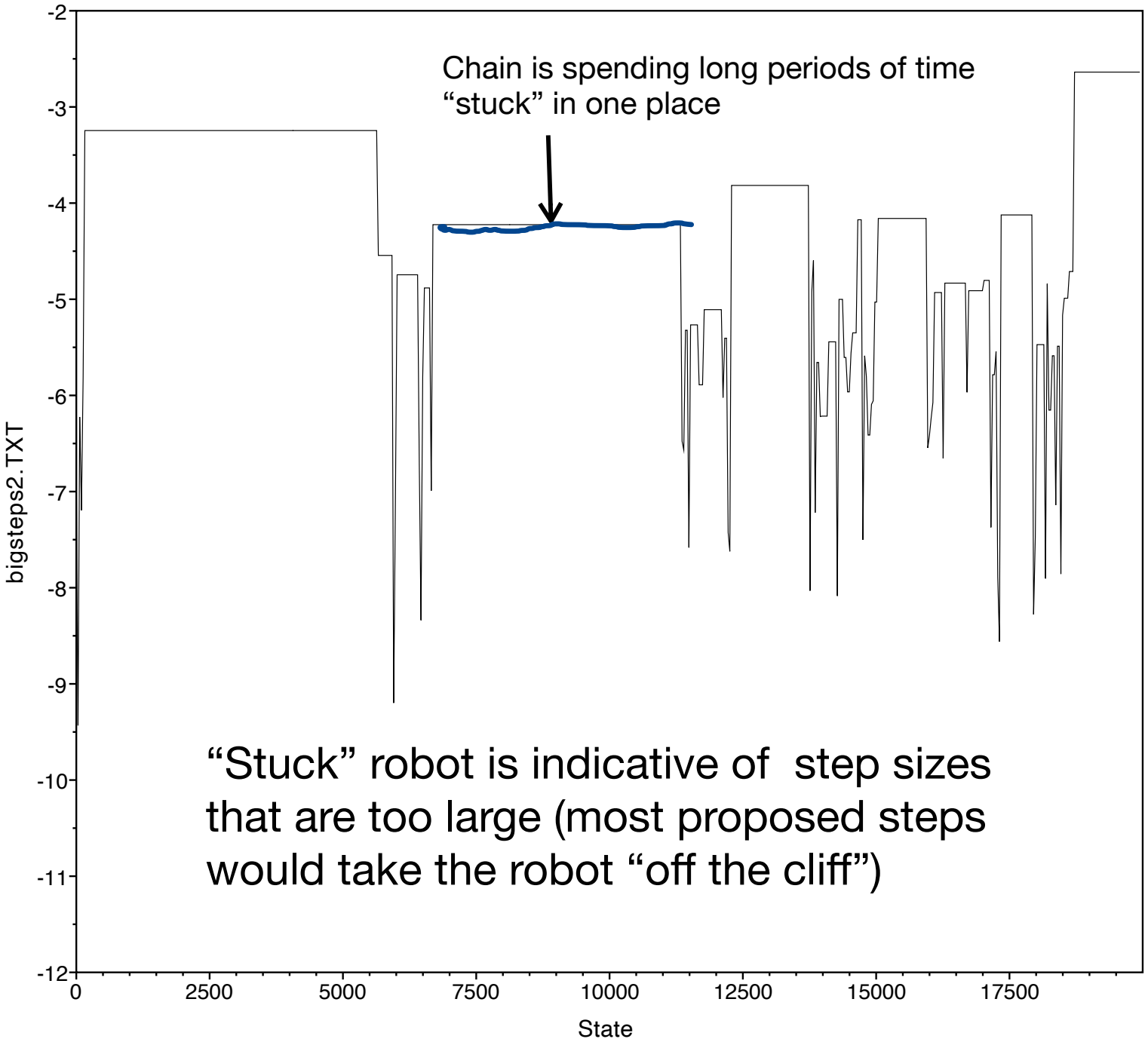
Target vs. Proposal Distributions

Proposal distributions with larger variance...



Disadvantage: robot often proposes a step that would take it off a cliff, and refuses to move

Advantage: robot can potentially cover a lot of ground quickly


Chain is spending long periods of time
“stuck” in one place
“Stuck” robot is indicative of step sizes that are too large (most proposed steps would take the robot “off the cliff”)
bigsteps2.TXT
State
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
0
2500
5000
7500
10000
12500
15000
17500

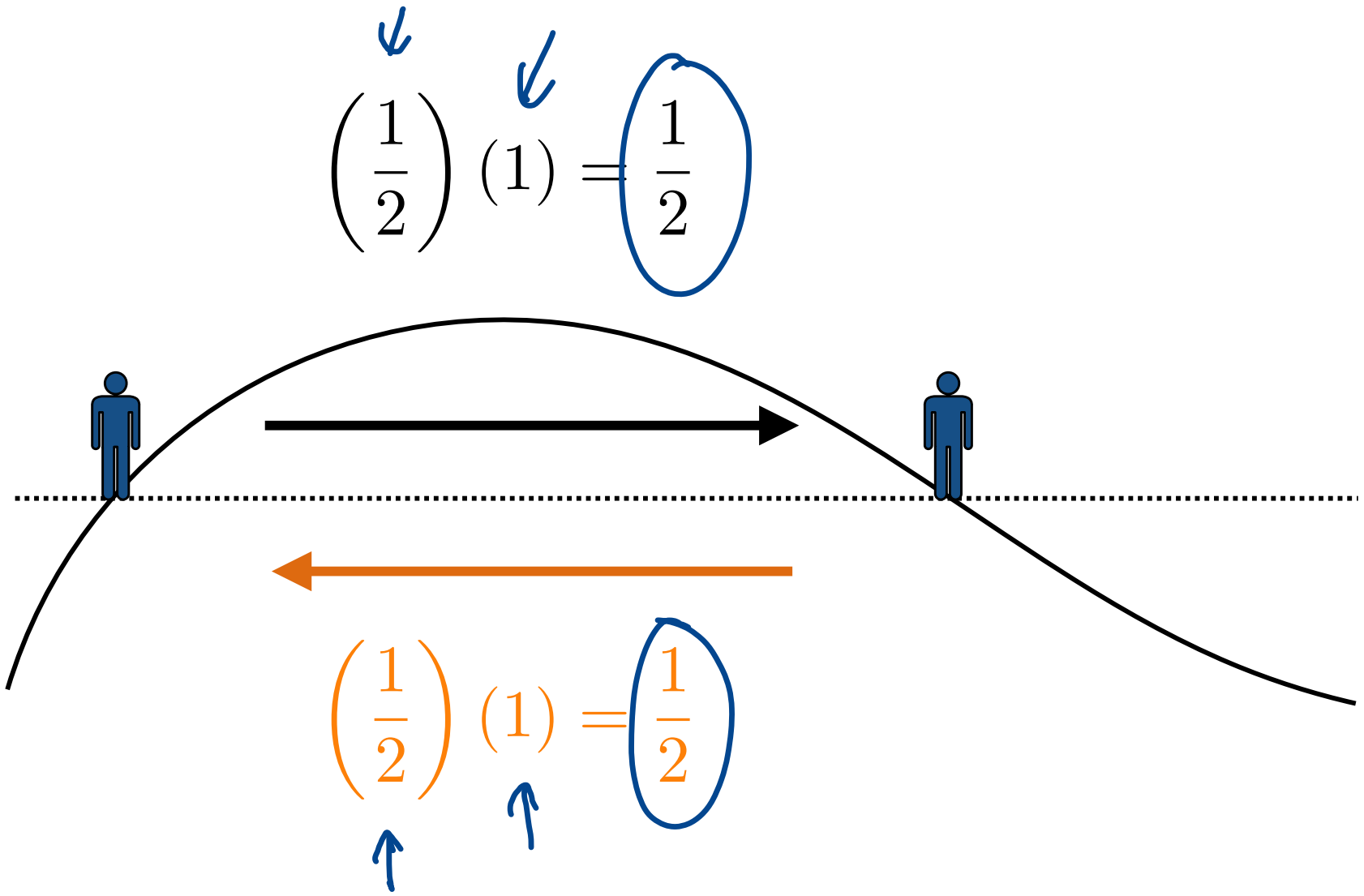
Metropolis Algorithm

$$R = \min \left\{ \frac{p(D|\theta^*)p(\theta^*)}{p(D|\theta)p(\theta)}, 1 \right\}$$

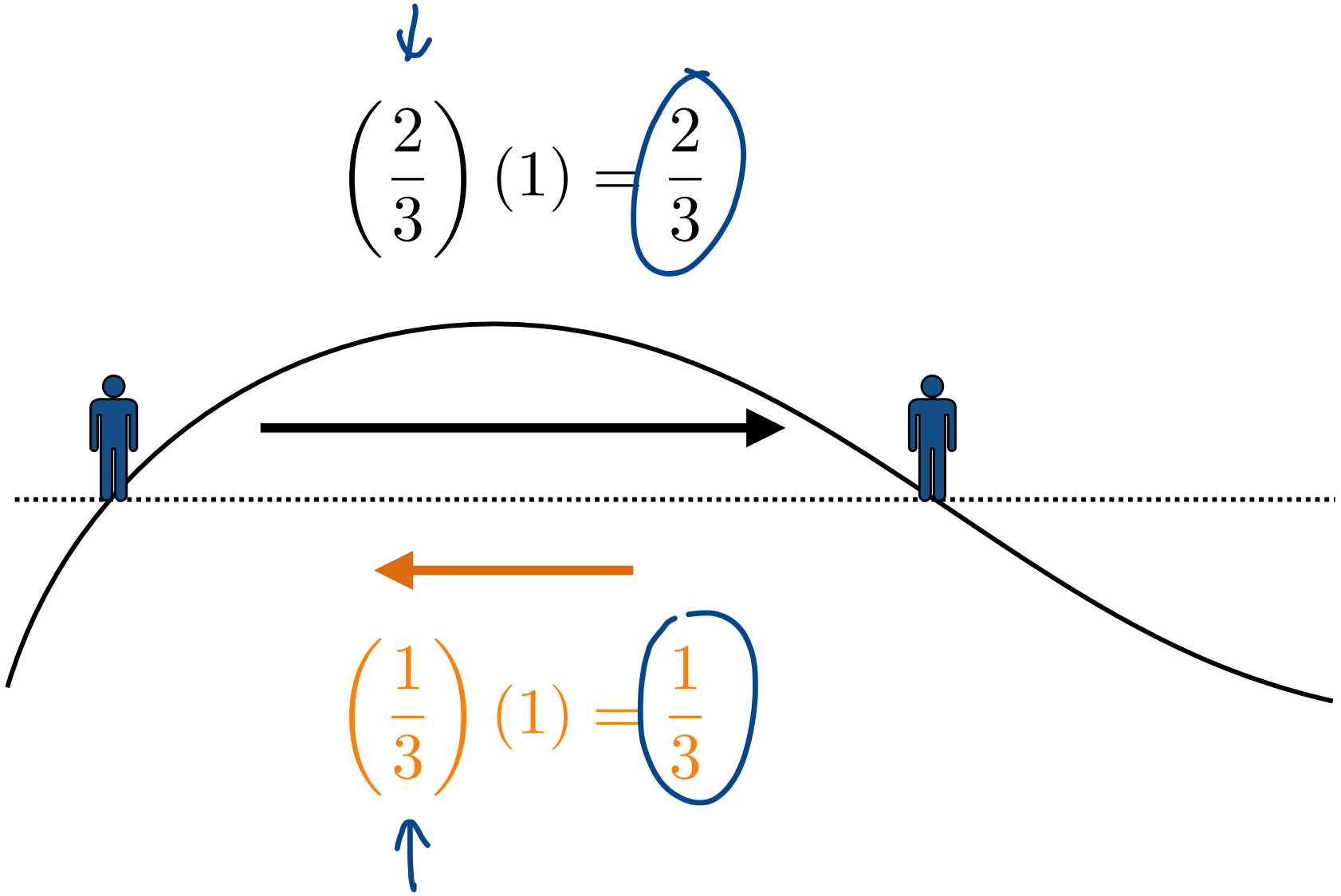
Posterior ratio

Metropolis et al. (1953)

Hastings ratio



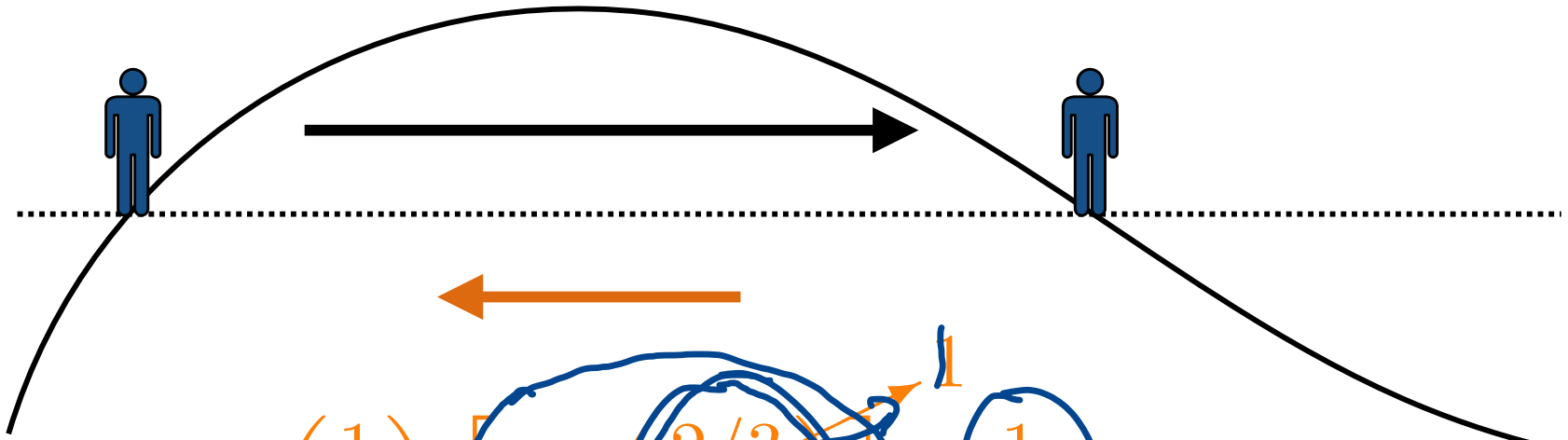
Hastings ratio



Hastings ratio

Metropolis-Hastings algorithm

$$\left(\frac{2}{3}\right)\left[(1)\left(\frac{1/3}{2/3}\right)\right] = \frac{1}{3}$$



$$\left(\frac{1}{3}\right)\left[(1)\left(\frac{2/3}{1/3}\right)\right] = \frac{1}{3}$$

Metropolis-Hastings Algorithm

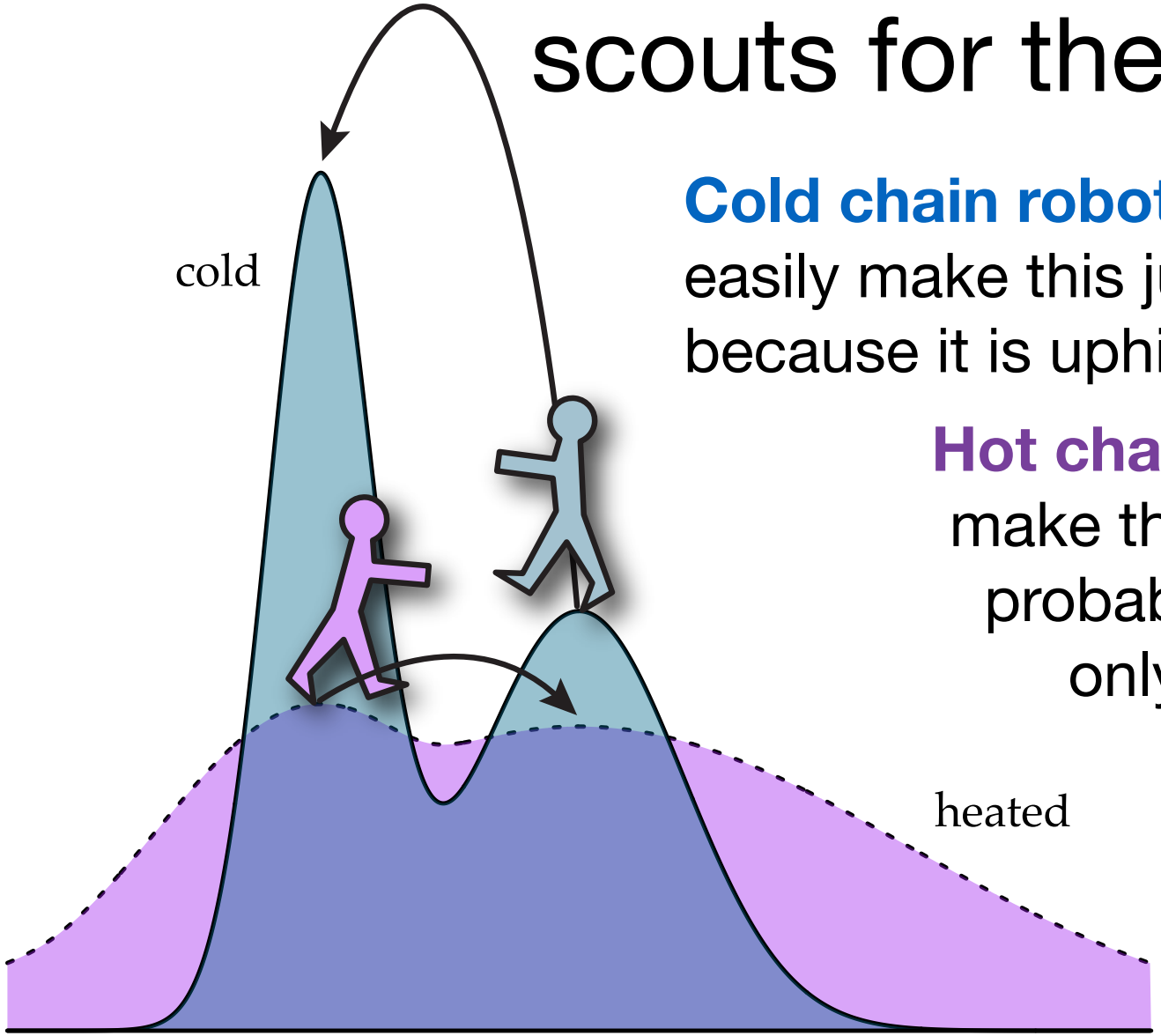
$$R = \min\left\{ \left[\frac{p(D|\theta^*)p(\theta^*)}{p(D|\theta)p(\theta)} \right] \left[\frac{q(\theta|\theta^*)}{q(\theta^*|\theta)} \right], 1 \right\}$$

Posterior ratio

Hastings ratio

Hastings (1970)

Heated chains act as scouts for the cold chain



Cold chain robot can easily make this jump because it is uphill

Hot chain robot can also make this jump with high probability because it is only slightly downhill

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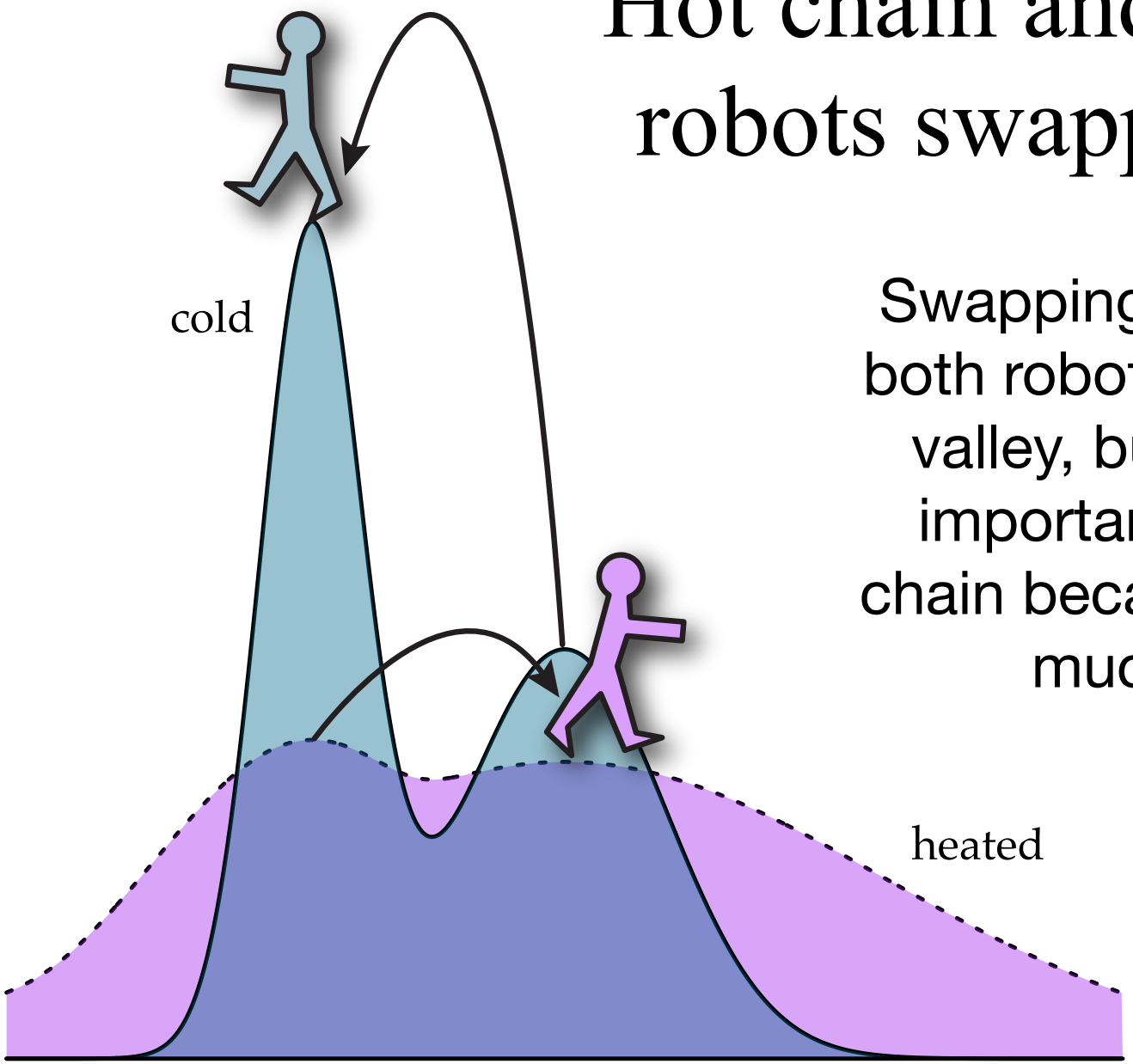
Guyer (1991)

Hot chain and cold chain robots swapping places



Swapping places means both robots can cross the valley, but this is more important for the cold chain because its valley is much deeper